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INFORMATION REDUCTION DUE TO CORRELATION

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In combining bodies of information, if some of the data is common to both sets, then the sets are correlated, and the potential amount of information is diminished. In some situations, the problem of correlation is solved by sampling at distances over which the correlation is negligible. This is not always possible. For correlated samples, we would like to know how much information can be extracted, what is a good sampling procedure, and what benefits extending the sampling window may bring. The problem of estimating the mean of a set of correlated data is considered.

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1. INTRODUCTION

In combining bodies of information, if some of the data is common to both sets, then the sets are correlated, and the potential amount of information is diminished as a result of the correlation. For example, when data is expensive it is common for one set of data to serve as the basis for many studies. Combining the results of these studies as if they were independent could lead to ill-founded confidence intervals for the final estimator. In some situations the measurements may be correlated. Pollution measurements of a body of water will be correlated across both time and location. Taking many water samples (as opposed to one) at a given location and time does not necessarily increase the information.

In many instances, there is a need to extract information from data that is self-correlated. In some situations the problem of correlation is solved by sampling at distances over which the correlation is considered negligible. This is an option when the size of the sampling window can be controlled by the designer. In other situations, there is a tradeoff between the size of the sampling window and cost; thus, it is useful to have a method available to gain insight into these tradeoffs. It is the purpose of this report to provide some insight and clarify some of the issues associated with this problem.

2. BACKGROUND

In signal processing the amount of information that can be extracted from a signal is a function of the variance, the number of samples taken, the correlation of the signal, and the observation interval. When the correlation time of the signal is longer than the sampling window, it is possible for an estimate to contain a large bias. This situation can arise when an incoming projectile is detected a short distance from its intended target and the autocorrelation time of the measurement noise is longer than the time remaining until impact. For certain noise functions we would like to know how much information can be extracted from the signal, what is a good sampling rate, and how much gain is there in extending the detection distance (or increasing the observation window). Usually the dimension of correlation is time; however, it can be proximity along any dimension. If the correlation is considered a nuisance, an adequate model for the expectation of two observations is an exponential correlation model (Young and Jakeman 1979;

Seber and Wild 1989). If v is the variable of correlation, then β is the parameter in the correlation function. The equation,

$$Cor(v_i, v_j) = e^{-\beta d(v_i, v_j)}, \tag{1}$$

gives the correlation between observations separated by $d(v_i, v_j)$ where d is the appropriate distance function. Typically, variables are correlated across time or physical location.

When it is not feasible to partition the sample space into groups or clusters that are not highly correlated, the effects of correlation must be addressed. An approach to this issue is to consider the tradeoffs between the cost of an observation and the gain of information due to the observation. The gain of information is indicated by the reduction of the covariance. Thus, if an observation leads to a significant reduction of the covariance of the estimate then it is cost effective. One approach to this question is to find the reduction in variance for different sampling methods and then look at the performance/cost questions. The problem of estimating the mean of a set of correlated data is the problem of consideration.

3. THE MEAN AS AN ESTIMATOR

Consider the problem of estimating the mean over a fixed time interval when the measurement noise is correlated across time. The goal is to quantify the amount of information extracted for different sampling rates. As the sampling rate increases, the correlation between the obs. rvations will increase. The discussion focuses on the decrease in the variance as a function of increase in sampling rate. The problem stated mathematically follows. Find Var($\overline{Y}$) where

$$\bar{Y} = \sum_{i=1}^{N} \frac{1}{N} Y_i ,$$

$$Y_i = Y + V_i , \quad V_i \sim N(0, \sigma^2) ,$$

$$Cor(V_i, V_j) = e^{-\beta |t_j - t_i|} . \tag{2}$$

Let the total amount of time available be T, and assume that equally spaced observations will be taken. The correlation between successive observations, α, is defined by the following equation

$$\alpha = e^{\frac{-\beta T}{N-1}} \tag{3}$$

Correlation implies that the same thing is being measured on separate occasions, and thus reduces the potential information in a sample.

For uncorrelated observations, the inner product associated with the measurements is $I_N\sigma^2$. For simplicity, σ will be assumed to be one for the rest of the discussion. Let $\vec{X}$ be an N dimensional vector of ones and $\vec{Y}$ be the vector of observations; then the estimator of the average of Y_i is

$$\bar{Y} = \frac{1}{N}\vec{X}'\vec{Y}$$

and

$$Var(\bar{Y}) = \frac{1}{N}\vec{X}'I_N\vec{X}\frac{1}{N} = \frac{1}{N}\,. \tag{4}$$

When correlation across the observations exists, the observations are not independent. In this situation the covariance matrix of the observations is found by using Equation 3. Since α is the correlation between observations one time step from each other, the correlation between Y_i and Y_j is $\alpha^{|i-j|}$. The correlation matrix, Σ, associated with the set of measurements is

$$
\begin{matrix}
1 & \alpha & \alpha^2 & \dots & \alpha^{(N-1)} \\
\alpha & 1 & \alpha & \dots & \alpha^{(N-2)} \\
\alpha^2 & \alpha & 1 & \dots & \alpha^{(N-3)} \\
\alpha^3 & \alpha^2 & \alpha & & \alpha^{(N-4)} \\
. & . & . & & . \\
. & . & . & & . \\
. & . & . & & . \\
\alpha^{(N-1)} & \alpha^{(N-2)} & \alpha^{(N-3)} & \dots & 1
\end{matrix}
$$

As α approaches 1, the correlation matrix ceases to be positive definite. One way to see this idea is to consider sweeping the matrix on the (1,1) element. The sweep operator can be thought of as a variation of the Gram-Schmidt process. When the matrix is swept on the (1,1) element, the first row becomes orthogonal to the space spanned by the remaining altered vectors. In terms of inner products, the projection of the first observation onto another observation is removed from each observation in turn. This operation removes all the information contained in the first observation from the others. If each row contains values close to 1, the result of this will be that there is very little information left after the first row is processed. For a discussion of the sweep operator and its implementation see Dempster (1969) or Seber (1977).

Var($\overline{Y}$), with Σ as the covariance of the observations, is $\frac{1}{N}\vec{X}'\Sigma\vec{X}\frac{1}{N}$. Since $\vec{X}$ is a column of ones, this operation adds the values of each column and then adds the columns and divides the result by N^2. In this case, to find the variance of the estimate, the elements of the matrix are added and then that sum is divided by the square of the number of observations. Examination of the matrix shows there is one diagonal of ones of length N, there are two diagonals of length N-1 filled with α, two diagonals of length N-2 filled with α^2 and so on. Therefore,

$$
Var(\overline{Y}) = \frac{1}{N^2}\left(N + 2\sum_{i=1}^{N-1}(N-i)\alpha^i\right). \tag{5}
$$

If α is 0 then the formula does indeed reduce to the previous case of no correlation; and if α is 1 there is no reduction in the variance of the estimate by taking more data. If α=1 then

$$N + 2\sum_{i=1}^{N-1}(N-i) = N + 2\sum_{i=1}^{N-1} i$$

$$= N^2 \tag{6}$$

and using Equation 5 the $Var(\overline{Y}) = 1$. The value of α depends on β, the time period, T, and the number of observations, N. Rewriting Equation 5 to reflect this dependence yields,

$$Var(\bar{Y}) = \frac{1}{N^2}\left(N + 2\sum_{i=1}^{N-1}(N-i)e^{\frac{-\beta T i}{(N-1)}}\right) \tag{7}$$

as the formula of interest.

Using Equation 7, the $Var(\overline{Y})$ can be calculated from known values of T, β, and N. In this case, it is possible to reduce the number of variables by expressing T in terms of β. When β is small, the correlation will fall off slowly over time. In discussing different processes, the correlation times of the processes are typically compared. The correlation time of the process is defined as $\frac{1}{\beta}$. For the remainder of this report the signal length or sampling window will be in correlation time units. Using these time units, Equation 7 can be calculated from two variables—the number of correlation time units and the number of observations. In evaluating this formula, the result will indicate the reduction of uncertainty for a sampling window length, in correlation time units, and a given number of observations within the sampling window. The next task is to evaluate this formula at some interesting points and make some observations about the behavior of the $Var(\overline{Y})$ as T and N change. Table 1 shows $Var(\overline{Y})$ evaluated at the indicated values of N and T.

Table 1 shows that there is an optimal sampling rate for the calculation of the mean. The increase in the variance as a result of oversampling seems to approach its maximum value when the sampling rate is 512 observations per correlation time. This can be seen in row 1 of the table, when the signal duration is one-eighth of the correlation time, an increase in the number

Table 1. $Var(\overline{Y})$ at Values of T and N.

Sampling Interval	Number of Observations (N)							
1/β Time Units (T)	2	4	8	16	32	64	128	256
.125	.941	.950	.954	.957	.958	.959	.959	.959
.25	.889	.904	.913	.917	.919	.920	.921	.921
.5	.803	.822	.837	.844	.848	.850	.851	.852
.75	.736	.753	.770	.780	.785	.788	.789	.790
1.0	.684	.693	.712	.723	.729	.733	.734	.735
1.5	.611	.597	.615	.628	.635	.639	.641	.642
2.0	.568	.525	.539	.552	.560	.563	.566	.567
3.0	.525	.428	.430	.440	.447	.451	.453	.454
4.0	.509	.369	.357	.364	.370	.373	.375	.376
5.0	.503	.331	.306	.309	.314	.317	.319	.320

of samples from 64 does not increase the variance of the mean. Equation 7 was used to find the value of N that corresponded to the minimum variance for sampling intervals of 1 to 10 correlation units; these are displayed in Table 2 along with the variance obtained when the number of observations is twice the sampling interval. Examination of these values indicate that a good rule for selecting the optimal number of observations is: pick two if the signal has a length of less than one correlation time unit; otherwise set the number of samples to twice the number of correlation time units in the sampling window.

Oversampling, when using the mean as an estimator, inflates the variance of the estimate by up to 7% of its minimum. The consequences of setting the sampling rate too low are much worse than those associated with a high sampling rate. For correlated data, the mean is not the best estimate except for the case of two observations. Thompson (1991) discusses a method to find the best weighing factor to associate with correlated observations. Using the optimal weights, the variance will decrease as a function of N, unlike the equal weight case presented in Table 1 and Table 2.

4. OPTIMAL WEIGHTS

Optimal weights are those that yield a minimum variance unbiased linear estimate. The weight for an observation is inversely proportional to the variance associated with an observation

Table 2. Minimum Variance of Equation 7

Time	Minimum N	Variance	N = 2*Time	Variance
1	2	.6839	2	.6839
2	4	.5253	4	.5253
3	5	.4256	6	.4264
4	7	.3567	8	.3572
5	9	.3061	10	.3063
6	12	.2676	12	.2676
7	14	.2373	14	.2373
8	17	.2130	16	.2131
9	20	.1932	18	.1932
10	24	.1766	20	.1767

if the observations are independent; thus, if the variances associated with each observation are the same, the weights will be equal and the mean will be the optimal estimator. Each weight indicates the relative value of each observation. A more formal statement of this is: the optimal weights define the inner product that minimizes the error when the observations are projected onto a set of independent variables. Each weight is the Lagrange multiplier associated with the observation.

The following discussion assumes a fixed sampling window with the first two observations taken at the extremes of the interval. The intent is to demonstrate that for highly correlated observations an increase in the sampling rate may not result in a meaningful decrease of the variance; thus taking additional observations may not increase the useful information in a sample. Since all pairwise correlations must be considered, it is parsimonious to consider the change from two to three observations. The effects of taking an additional sample will be further diminished if the sample is also correlated with more distant neighbors; however, the ideas for analysis are the same as those in the change from two to three observations.

When using the optimal estimator, the addition of a data point always reduces the variance of the estimator. To illustrate this, the change in the variance of the optimal estimator will be

investigated when N goes from 2 to 3. A decrease indicates that more information can be extracted from any subinterval given the optimal weights. Assuming a correlation of α^2 between two observations, each has the same weight (.5). The variance is given by Thompson (1991) as

$$Var(\hat{Y}) = \frac{1+\alpha^2}{2} \quad . \tag{8}$$

Next, assume that an additional observation had been taken at the midpoint of the interval. The formulas for finding the three optimal weights are given by Thompson. Using the formula for three correlated observations, let $\sigma_1 = \sigma_2 = \sigma_3 = 1$, $\rho_{12} = \rho_{23} = \alpha$, and $\rho_{13} = \alpha^2$.

$$k_1 + k_2 + k_3 = 1$$

$$\begin{bmatrix} 2(1-\alpha^2) & 1-\alpha^2 \\ 1-\alpha^2 & 2(1-\alpha) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 1-\alpha^2 \\ 1-\alpha \end{bmatrix} \quad . \tag{9}$$

Solving Equation 9 leads to the following values,

$$k_1 = \frac{1}{3-\alpha}$$

$$k_2 = \frac{1-\alpha}{3-\alpha} \quad . \tag{10}$$

$$k_3 = \frac{1}{3-\alpha}$$

The value of each observation is proportional to the weight associated with it. The added observation, k_2, will have a small value when α is close to 1. The following formula gives the variance of the estimator.

$$Var(\hat{Y}) = k_1^2 + k_2^2 + k_3^2 + 2k_1k_2\alpha + 2k_2k_3\alpha + 2k_1k_3\alpha^2 \quad . \tag{11}$$

Using this formula with the values of the weights plugged in gives

$$Var(\hat{Y}) = \frac{2+(1-\alpha)^2 + 4(1-\alpha)\alpha + 2\alpha^2}{(3-\alpha)^2}$$

$$= \frac{1+a}{3-a} \quad . \tag{12}$$

As α approaches 1, $Var(\overline{Y})$ goes to 1. The addition of an observation that is highly correlated with its neighbors seems pointless as it will have very little influence on the estimate and will not significantly increase the information extracted from the signal. The exact amount of the decrease in the variance can be found by subtracting the right side of Equation 12 from the right side of Equation 8. This is done in the following:

$$\frac{1+\alpha^2}{2} - \frac{1+\alpha}{3-\alpha} = \frac{(1+\alpha^2)(3-\alpha) - 2(1+\alpha)}{2(3-\alpha)}$$

$$= \frac{(1-a)^3}{2(3-\alpha)} \quad . \tag{13}$$

Since both the numerator and denominator are positive for $0 < \alpha < 1$, the information gain is positive. If α is 0.9, the gain due to the extra observation is .000238; for an α of 0.5 the gain would be 0.025. Correlated errors drastically reduce the information contained in the set of observations.

Finding the optimal weights for a set of observations involves finding the inverse of a matrix that is ill-conditioned if the correlation is high. In many cases, it may be numerically impossible to perform this operation. Optimal weights can be found using the method discussed in Case 7 of Thompson (1991). The only reasonable way to calculate the weights is using software for matrix operations. A program was devised to evaluate the variance when optimal weights are used for estimation; the length of the sampling window and number of observations were varied. The results are shown in Table 3.

Table 3. Variance of Optimal Estimator at Values of T and N

Sampling Interval	Number of Observations (N)				
1/β Time Units (T)	2	3	4	5	10
0.5	.8033	.8008	.8004	.8002	.8000
1	.6839	.6712	.6687	.6678	.6669
2	.5677	.5197	.5090	.5051	.5010
3	.5249	.4405	.4191	.4109	.4022
4	.5092	.3963	.3639	.3511	.3370
5	.5034	.3708	.3282	.3107	.2909
10	.5000	.3363	.2636	.2276	.1804

First consider each column. The reciprocal of the number of observations is the lower bound for each column; it is the variance that would be obtained if the observations were independent. Moving down a column shows the effects of increasing the duration of the sampling window. Moving across a row shows the effects of adding more observations to a fixed sampling interval. To complement the values in the table, two additional cases were evaluated: a sample window of 10 correlation units with 20 observations results in a variance of 0.1698; and, for the same window, 30 observations result in a variance of 0.1680. As the number of observations are increased, the variance seems to decrease asymptotically. As a general rule, the sampling rate should be set to two observations per correlation time.

In actual situations, the size of the observation window in correlation time units and the intensity of noise correlation may not be precisely known, or either may vary over time. Typically, it is impossible to precisely calculate the optimal weights beforehand. The amount of information available for processing is directly proportional to the number of correlation time units over which the observations are made; rather than the number of samples.

5. CONCLUSIONS

When the interval to collect data is small, the possibility of correlated data should be addressed. If the observations are correlated, the length of the sampling window may need to be increased. For an active protection system sensor this indicates that extending the initial detection range has the greatest potential to increase the accuracy of the system if the

observations are correlated. The prudent approach would be to gather as much information as possible on the noise processes that will degrade a sensor system's performance and then decide how many correlation time units are needed for adequate performance. This demands knowledge of the energy transformations associated with a specific sensor, and the effects of the atmosphere or propagation media on the signal. The amount of correlation and intensity of the sensor noise process will limit the information that can be extracted over a short time period.

If the formulas for uncorrelated observations are used, when the observations are correlated, the variance estimate will be too low. In effect, there are fewer independent observations. Simulation can be used to assess the effects of specific noise processes. When using least squares estimation, the covariance estimate of the parameters will be deflated if the sampling rate introduces correlation. When the observations are correlated, an extension of the observation window is the most effective way to increase performance.

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